



# Market based transmission expansion and reactive power planning with consideration of wind and load uncertainties



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## ABSTRACT

In this paper, a new methodology for Transmission Expansion Planning (TEP) in deregulated electricity market is presented. The proposed TEP is associated with Reactive Power Planning (RPP), reliability assessment and also consideration of wind and load uncertainties. The proposed planning aims at investment cost minimization, social welfare maximization and satisfying reliability constraint at the same time with taking into account wind and load uncertainties. Expected Energy Not Supplied (EENS) is used as an index for reliability evaluation. At first, Monte-Carlo simulation is used to obtain the Probability Density Function (PDF) of Wind Turbine Generator (WTG) output. Then, the WTG and load uncertainties are considered in TEP formulation. Particle Swarm Optimization (PSO) method is considered to solve the proposed planning problem which is a constrained nonlinear mixed integer optimization programming. Simulation results on two standard test systems (Garver and RTS systems) verify the effectiveness of the proposed planning for consideration of wind and load uncertainties in TEP problem under electricity market environment. Also, the proposed method leads to reduction of the total investment cost, the reliability improvement and the social welfare maximization.

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## Nomenclature

### Symbols and Indexes

$b_{ij}^{sh}$	shunt susceptance of line or transformer $ij$ (if $ij$ is a transformer $b_{ij}^{sh} = 0$ ) (p.u.)
$b_i^{sh}$	shunt susceptance at bus $i$ (p.u.)
$c_{0k}$ and $c_{1k}$	installation costs and unit costs for a VAR-plant at bus $k$ (\$)
$c_1$ and $c_2$	learning factors
$f_1(q, u)$	cost function of locally reactive sources (\$)
$f_2(P_S, P_L, \Pi_L, \Pi_S)$	cost function of social benefit (\$/h)
$g_{best}(t)$	global best value
$g_{ij}$ and $b_{ij}$	conductance and susceptance of the transmission line or transformer $ij$ (p.u.)
$i, j$	bus indices
$N_B$	set of all buses
$n_j$	number of new added transmission lines to branch $j$
$n^{max}$	maximum number of new added lines to each branch
$N_s$	number of scenarios
$n_t$	project life-time
$P_{best}(t)$	$i^{th}$ particle best solution
$p_{id}(t)$	position of the $d^{th}$ dimension of the $i^{th}$ particle in $t^{th}$ iteration
$P_G^{max}$ and $Q_G^{max}$	maximum limit of real and reactive power generation limits (p.u.)
$P_G^{min}$ and $Q_G^{min}$	minimum limit of real and reactive power generation limits (p.u.)
$q$	total amount of locally reactive sources (MVar)
$q_k$	amount of locally reactive source in bus $k$ (MVar)
$q^{max}$ and $q^{min}$	maximum and minimum amounts of reactive sources (p.u.)
$r_t$	discount rate
$S^{from}$ and $S^{to}$	apparent power flow through the branches in both terminals (p.u.)
$S^{max}$	apparent power flow limits (MVA)
$\nu_0$	total investment due to the addition of new circuits (\$)
$\nu_1$	total investment of locally reactive sources (\$)
$\nu_2$	total cost of social welfare (\$/h)
$\nu_{annual}$	annual cost of the proposed planning problem (\$/year)
$\nu_{id}(t)$	velocity of the $d^{th}$ dimension of the $i^{th}$ particle in $t^{th}$ iteration
$V^{max}$ and $V^{min}$	maximum and minimum voltage magnitudes (p.u.)
$\nu_{total}$	total cost of the proposed planning problem (\$)
$w$	inertia factor

$\Omega_1$	set of all load buses
$k \in \Omega_1$	$k^{th}$ load bus

### Vectors and matrixes

$c$	lines cost vector (\$)
$n$	added lines vector
$N$	matrices containing the new lines
$N_0$	matrices containing the existing lines
$P_D$ and $Q_D$	real and reactive power demand vectors (MW and MVar)
$P_G$ and $Q_G$	real and reactive power generation vectors (MW and MVar)
$P_{Si}$ and $P_{Li}$	vectors of supply and demand powers in bus $i$ (MWh)
$u$	binary vector that indicates whether or not to install reactive power sources
$u_k$	binary vector that indicates whether or not to install reactive power sources at bus $k$
$V$ and $\Theta$	magnitude and angle of voltages vectors (p.u. and radian)
$\Pi_{Si}$ and $\Pi_{Li}$	vectors of supply and demand bids in bus $i$ (\$/MWh)

### Abbreviations

DG	distributed generation
EENS	expected energy not supplied
EENS <sup>max</sup>	maximum amount of expected energy not supplied
FOR	forced outage ratio
GA	genetic algorithms
ISO	independent system operator
LMP	local marginal prices
LOLE	loss of load expectation
LOLE <sup>max</sup>	maximum amount of loss of load expectation
MCS	Monte-Carlo simulation
MTTR	minimum time to repair
OPF	optimal power flow
PDF	probability distribution function
PSO	particle swarm optimization
RPP	reactive power planning
RTS	reliability test system
TEP	transmission expansion planning
WTG	wind turbine generator

## 1. Introduction

TEP problem aims at expanding the power system transmission network to serve the growing demand in the future. The TEP problem denotes where and when, new lines should be installed in the network to support the customers demand. The TEP problem in regulated power systems intends to minimize the investment cost, but in the deregulated electricity markets, the TEP plans to provide a competitive environment for all participants.

TEP problem is a nonlinear mixed integer constrained programming. The commonly used models for TEP problem modeling are DC [1] and AC models [2,3]. Also, many different methods have been applied to solve the TEP problem. These approaches are divided into mathematical and Meta-heuristic approaches. The mathematical methods such as Linear programming [4], nonlinear programming [5], mixed integer programming [6] and Bender's decomposition [7] have been carried out for solving the TEP

problem. Also, some heuristic optimization methods such as PSO [8], Tabu Search [9], Harmony Search [10], Genetic Algorithms [11], Decimal Coded Genetic Algorithms [12], Genetic Algorithms-based quadratic programming [13], Chaotic [14], Ant Colony [15] and Differential Evolution [16] have been successfully used for solving the TEP problem.

From another view, in TEP problem, many different parameters have been considered as objective function. The minimization of the investment cost is a conventional objective function for TEP problem [2]. Transmission surplus capacity as an objective function for TEP problem has been investigated in [14]. The maximization of the transmission reliability and the minimization of the investment cost are considered as objective functions for TEP problem in [15]. Combination of TEP problem with RPP is presented in [17]. This paper has shown the TEP problem associated with RPP results in fewer new transmission lines to be installed in comparison with the conventional TEP method.

In deregulated electricity market the objective functions are different from the conventional markets. In deregulated markets, the market concepts such as congestion surplus, social welfare and nodal prices are considered in the TEP problem. Considering the investment cost and the annual generation cost in the TEP problem under electricity market has been reported in [16]. In [17] the combination of TEP and RPP has been presented in electricity market. Coordination of transmission and generation capacity planning in a deregulated market has been investigated in [18]. A bi-level method to TEP under market environment has been studied in [19]. A market oriented TEP problem with considering the social welfare and investment costs has been reported in [4].

Recently, DGs such as wind farms are rapidly developed in power systems, because of their favorable characteristics [20]. However, wind power output is continuously changing and thus, an extra factor of uncertainties is introduced in power system operation and planning. Also, the loads in power system are uncertain. Therefore, in power systems with wind power and load uncertainties, the deterministic TEP methods are not suitable. A method with considering the wind power and load uncertainties should be incorporated [21].

In this paper, the AC-TEP associated with RPP is considered. The proposed planning is carried out in a deregulated electricity market with taking into account wind and load uncertainties. The proposed planning aims at the investment cost minimization and the social welfare maximization by considering reliability index at the same time. The reliability of planning is evaluated by using EENS index, which is calculated by using Monte-Carlo simulation. Also, the wind and load uncertainties are incorporated by using Monte-Carlo simulation. Also, PSO algorithm is used to solve the problem. Simulation results, which are carried out on two typical power systems, show the validity of the proposed planning.

It should be noted that load uncertainties are modeled as normal Probability Distribution Function (PDF). The normal PDF is the most commonly used method to model different uncertainties in electric power systems such as load, cost, fuel etc.; it is also used in this paper [17–18,22–24].

## 2. Probabilistic WTG output model

Wind turbine output is nonlinearly related to the wind speed. With changing wind speed, WTG output may vary between zero to its rated output and, hence leads to fluctuations in power flow. As a result, all power system characteristics such as nodal prices, social welfare, line transmission powers change. Therefore, these uncertainties should be incorporated in power flow formulation. A probabilistic model is a very suitable model for tackling this problem. The commonly used model for wind speed is Weibull distributions, and the shape and scale parameters of the distributions can be derived from the mean and standard deviation of the wind speed [21]. The nonlinear model of a WTG relating the power output characteristics to the input wind speed has three

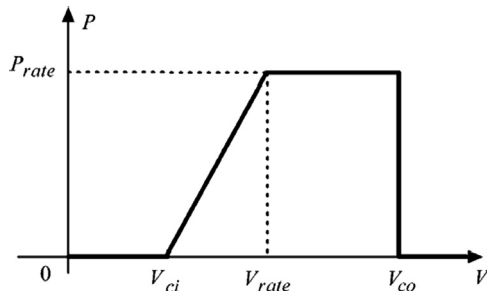


Fig. 1. Relationship between wind speed and WTG output power.

parameters namely cut in speed ( $V_{ci}$ ), rated speed ( $V_{rate}$ ) and cut out speed ( $V_{co}$ ). The output power of a WTG unit may be approximated by (1). The relationship between wind speed and WTG active power output is depicted in Fig. 1 [21].

$$P = \begin{cases} 0 & 0 \leq V \leq V_{ci} \\ P_{rate} \frac{(V - V_{ci})}{(V_{rate} - V_{ci})} & V_{ci} \leq V \leq V_{rate} \\ P_{rate} & V_{rate} \leq V \leq V_{co} \\ 0 & V_{co} \leq V \end{cases} \quad (1)$$

By using wind speed distribution and WTG characteristic, MCS can be applied to simulate the WTG output distribution. For example, for a 150 MW WTG unit, the WTG output probabilistic distribution is depicted in Fig. 2. In this figure, the WTG parameters are as  $V_{ci}=4$ ,  $V_{rate}=10$  and  $V_{co}=22$  m/s, wind speed mean 5.4 m/s and wind speed standard deviation 2.7 m/s [21]. The WTG output distribution in Fig. 2 can be described with the following function [21]

$$f(x) = \begin{cases} F_{zero} & x = 0 \\ g(x) & 0 \leq x \leq P_{rate} \\ F_{rate} & x = P_{rate} \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

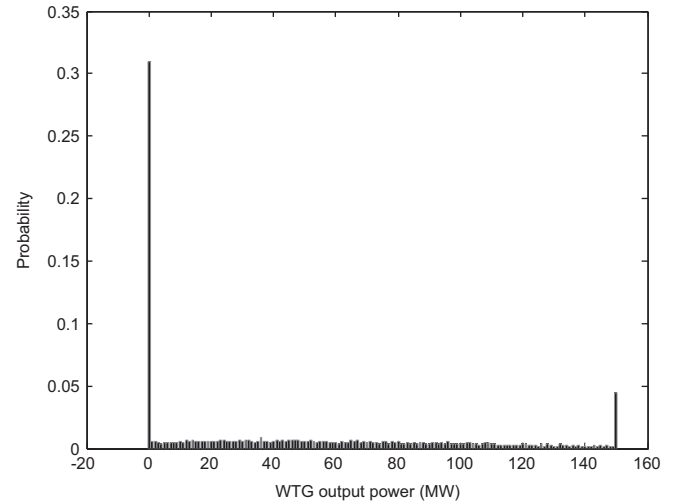


Fig. 2. Distribution of WTG output power.

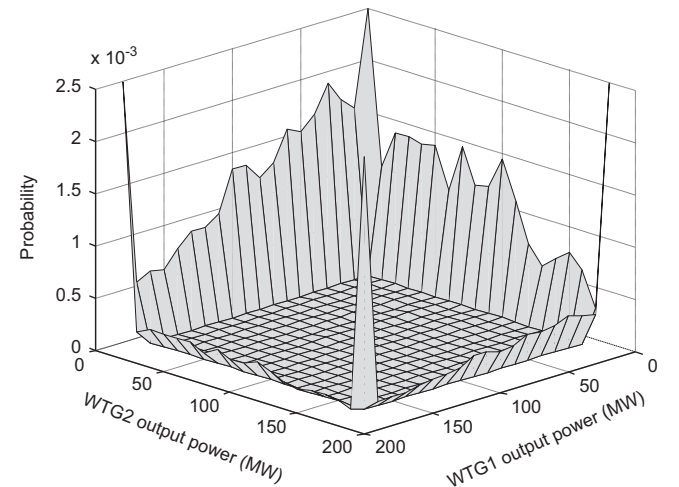


Fig. 3. Multi-dimension distribution of WTGs output power.

where function  $g(x)$  can either be expressed as a fitted polynomial function or be represented by discrete samples. Also, in the case of existence of two WTG units in a system, WTGs output distribution can be depicted as Fig. 3.

### 2.1. Power system analysis in the presence of uncertainties

In electric power systems installed with WTG, as mentioned in the previous section it is necessary to consider WTG uncertainties. Also, other kinds of uncertainties such as load uncertainties should be incorporated. Here, a general method to tackle with uncertainties is presented. The scenario based MCS is a commonly used and suitable tool for incorporating uncertainties in problem. The proposed algorithm as shown in Fig. 4 is a general method which can be used to tackle with any kind of uncertainties in power systems. At first, for each scenario of MCS, the wind speed and load are randomly generated based on their probability distribution. In this paper, the uncertainties of the load are modeled as a normal Probability Distribution Function (PDF). Then, the WTG output and its probability can be calculated by using (1) and (2) respectively. Also, the probability of the random load can be achieved by using its normal PDF. In this situation, the WTG and load are set on their randomly generated values.

By using the load probability and WTG output probability, the probability of the current scenario is also calculated. Then, an Optimal Power Flow (OPF) is carried out and the required outputs

such as line transmission powers, nodal prices, and social welfare are recorded. Thus, in each scenario, the variables and their probabilities are obtained and PDF can be easily calculated. The scenarios are iterated until a breaking criterion is met. By using the proposed method, the relationship between WTG output powers, the probability of WTG output powers and the system parameters are obtained. Fig. 5 shows the relationship between WTG output power and system congestion surplus in test case 1 (test systems are presented in section 5). Also, Fig. 6 shows the relationship between WTG output power and line 2–3 transmission power in test case 1. It is clearly seen that the WTG uncertainties affect the system parameters and there is a PDF for each parameter of system such as line transmission power or system congestion surplus. With regard to this issue, Power system planning in the presence of PDF for parameters is very complicated.

Fig. 7 shows the relationship between WTGs output power and system congestion surplus in test case 2. In this test case, there are two WTG units. It is seen from Figs. 5–7 that the WTG output has a great effect on the system parameters and this uncertainty should be incorporated in any planning. Thus, it is valuable to find a way

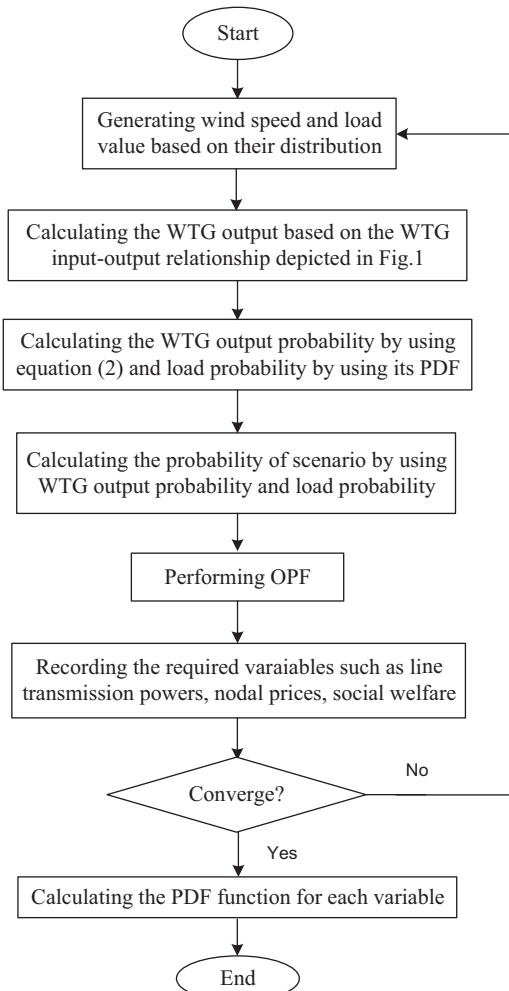


Fig. 4. Flowchart of generating PDF by using MCS.

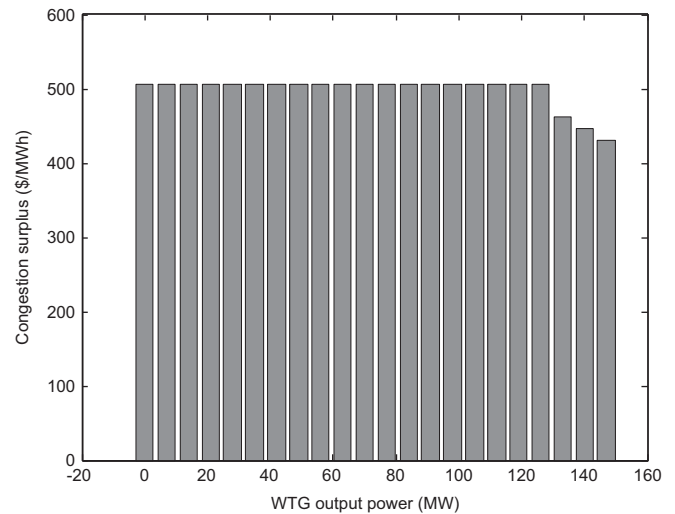


Fig. 5. Relationship between WTG output power and congestion surplus in test case 1.

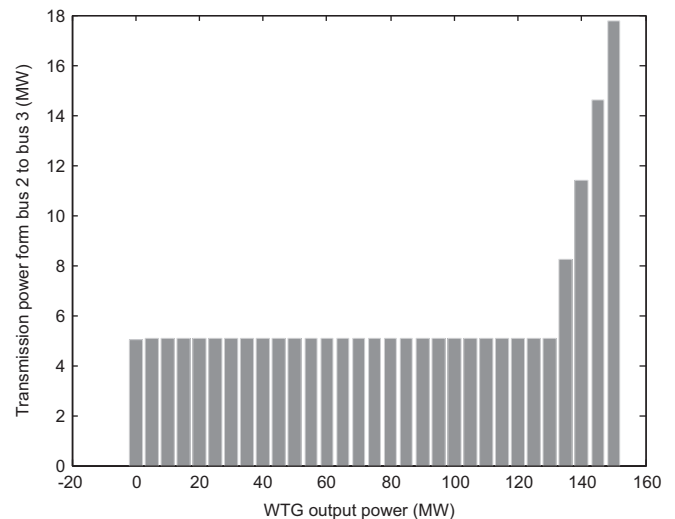
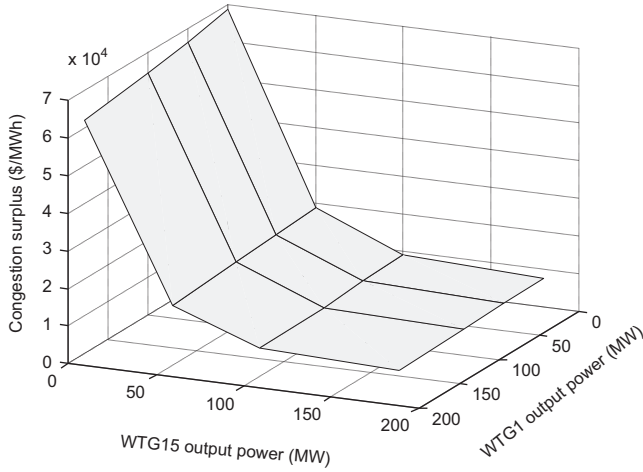


Fig. 6. Relationship between WTG output power and transmission power of line 2–3 in test case 1.



**Fig. 7.** Relationship between WTGs output power and congestion surplus in test case 2.

to convert the PDF function of a parameter to an equivalent fixed number. The next section presents a mathematical-probabilistic method to deal with this problem. It is worth mentioning that, Figs. 5–7 show the relationship between WTG output power and system parameters. But, Figs. 2 and 3 show the relationship between WTG output power and the probability. So, the relationship between system parameters and their probability (this relation is known as PDF) can be easily calculated. In fact, there is a correlation between WTG output power, the probability of WTG output power and any system parameters.

## 2.2. Calculation of expected value for parameters with PDF

Expected value (mean) is a very effective and commonly used method to convert a PDF to a number. It is the weighted average of all possible values of a random probabilistic variable. By calculating the expected value, not only the nature of uncertainty is considered, but also the calculations and simulations can be performed without extra complexity. The expected value of the variable  $X$  which takes value  $x_1$  with probability  $p_1$  is calculated as follows:

$$E(X) = \sum_{k=1}^n x_k p_k \quad (3)$$

Therefore, the system uncertainties can be eliminated and any PDF is converted to a number. The proposed method is used to include the parameters with PDF (such as market congestion surplus) into TEP problem with considering the wind power uncertainties.

## 3. Problem formulation

In electric power systems, technical and economical aspects are in conflict with each other. Therefore, considering one of them as objective function is not appropriate. Considering different objectives for TEP problem covers all aspects of planning. Thus, in this paper, both are considered. Technical objectives try to improve the power system operation, but economical objectives try to reduce the cost. In this paper, the reduction of the investment cost of lines and reactive sources are considered as economical objectives. Also, increasing social welfare of the network is considered as a technical objective function. These objective functions are defined as;

**Cost of TEP problem ( $v_0$ ):** the cost on new installed lines is computed as (4) [17].

$$v_0 = c^t n \quad (4)$$

**Cost of RPP problem ( $v_1$ ):** the cost of new locally reactive sources is computed as (5) [17].

$$v_1 = f_1(q, u) = \sum_{k \in \Omega_1} (c_{0k} + c_{1k} q_k) u_k \quad (5)$$

**Cost of social welfare ( $v_3$ ):** the social welfare of the system is obtained by (6) [25].

$$v_2 = f_2(P_S, P_L, \Pi_L, \Pi_S) = \sum (\Pi_{Li} \times P_{Li} - \sum \Pi_{Si} \times P_{Si}) \quad (6)$$

The objective function  $v_2$  shows the difference between the payment of consumers and the revenue of producers. This objective function is known as “congestion surplus” [25]. Besides, the minimization of congestion surplus is equal to the maximization of the social welfare and the two expressions are equivalent in meanings. Thus, this paper tries to maximize the social welfare.

The objectives  $v_0$  and  $v_1$  are per “\$”, but  $v_2$  is per “\$/h”. Thus, these units should be converted to the same unit. For easy comparison, all parameters are converted to “\$/year”. Therefore, the investment cost ( $v_0$  and  $v_1$ ) is converted to “\$/year” by (7) [26]. Also, the congestion surplus is converted to “\$/year” by (8).

$$v_{\text{annual}} = v_{\text{total}} \times \frac{r_t \times (1+r)^{n_t}}{(1+r)^{n_t} - 1} \frac{\$}{\text{year}} \quad (7)$$

$$v_2 \text{ annual} = v_2 \times 24 \times 365 \frac{\$}{\text{year}} \quad (8)$$

where, the coefficients “24” and “365” show the “hours of a day” and the “days of a year”, respectively. Eventually, the objective functions are rearranged as follows:

$$J_0 = v_0 \times \frac{r_t \times (1+r)^{n_t}}{(1+r)^{n_t} - 1} \frac{\$}{\text{year}} \quad (9)$$

$$J_1 = v_1 \times \frac{r_t \times (1+r)^{n_t}}{(1+r)^{n_t} - 1} \frac{\$}{\text{year}} \quad (10)$$

$$J_2 = v_2 \times 24 \times 365 \frac{\$}{\text{year}} \quad (11)$$

where the variable “year” shows 365 days,  $J_0$  demonstrates the annual cost of new installed lines,  $J_1$  demonstrates the annual cost of new locally reactive sources and  $J_2$  is the annual cost of congestion surplus of the system. These three parameters are considered as the objective function for TEP in the following form:

$$\text{Min } J_0 \quad (12)$$

$$\text{Min } J_1 \quad (13)$$

$$\text{Min } J_2 \quad (14)$$

Subject to,

$$P(V, \Theta, n) - P_G + P_D = 0 \quad (15)$$

$$Q(V, \Theta, n) - Q_G + Q_D - q = 0 \quad (16)$$

$$P_G^{\max} \leq P_G \leq P_G^{\max} \quad (17)$$

$$Q_G^{\min} \leq Q_G \leq Q_G^{\max} \quad (18)$$

$$V^{\min} \leq V \leq V^{\max} \quad (19)$$

$$(N + N_0) S^{\text{from}} \leq (N + N_0) S^{\text{max}} \quad (20)$$

$$(N + N_0) S^{\text{to}} \leq (N + N_0) S^{\text{max}} \quad (21)$$

$$q^{\min} \leq q \leq q^{\max} \quad (22)$$



$$0 \leq n \leq n^{\max} \quad (23)$$

$$\text{EENS} \leq \text{EENS}^{\max} \quad (24)$$

Constraints (15) and (16) show the power balance in buses. The bounds of generator output power are shown by (17) and (18). Constraint (19) shows the limits of voltage levels. The limits of transmission power in lines are presented by (20) and (21). Constraint (22) shows the limit for installation of reactive sources. Constraint (23) represents the limit of the new installed lines in each corridor and the reliability constraint is represented by (24). The parameters  $P(V, \Theta, n)$  and  $Q(V, \Theta, n)$  in (15) and (16) are computed as (25) and (26). Also, the bus admittance matrices ( $G$  and  $B$ ) are calculated as (27) and (28).

$$P_i(V, \Theta, n) = V_i \sum_{j \in N_B} V_j [G_{ij}(n) \cos \theta_{ij} + B_{ij}(n) \sin \theta_{ij}] \quad (25)$$

$$Q_i(V, \Theta, n) = V_i \sum_{j \in N_B} V_j [G_{ij}(n) \sin \theta_{ij} + B_{ij}(n) \cos \theta_{ij}] \quad (26)$$

$$G = \begin{cases} G_{ij}(n) = -(n_{ij}g_{ij} + n_{ij}^0g_{ij}^0) \\ G_{ji}(n) = \sum_{j \in \Omega_1} (n_{ij}g_{ij} + n_{ij}^0g_{ij}^0) \end{cases} \quad (27)$$

$$B = \begin{cases} B_{ij}(n) = -(n_{ij}b_{ij} + n_{ij}^0b_{ij}^0) \\ B_{ji}(n) = b_i^{sh} + \sum_{j \in \Omega_1} [n_{ij}(b_{ij} + b_{ij}^{sh}) + n_{ij}^0(b_{ij}^0 + (b_{ij}^{sh})^0)] \end{cases} \quad (28)$$

Eventually, the element ( $ij$ ) of  $S^{from}$  and  $S^{to}$  in (20) and (21) are calculated as (29) and (30).

$$S_{ij}^{from} = \sqrt{(P_{ij}^{from})^2 + (Q_{ij}^{from})^2} \quad (29)$$

$$S_{ij}^{to} = \sqrt{(P_{ij}^{to})^2 + (Q_{ij}^{to})^2} \quad (30)$$

where,

$$P_{ij}^{from} = V_j^2 g_{ij} - V_i V_j (g_{ij} \cos \theta_{ij} + b_{ij} \sin \theta_{ij}) \quad (31)$$

$$Q_{ij}^{from} = -V_i^2 (b_{ij}^{sh} + b_{ij}) - V_i V_j (g_{ij} \sin \theta_{ij} - b_{ij} \cos \theta_{ij}) \quad (32)$$

$$P_{ij}^{to} = V_j^2 g_{ij} - V_i V_j (g_{ij} \cos \theta_{ij} + b_{ij} \sin \theta_{ij}) \quad (33)$$

$$Q_{ij}^{to} = -V_j^2 (b_{ij}^{sh} + b_{ij}) - V_i V_j (g_{ij} \sin \theta_{ij} - b_{ij} \cos \theta_{ij}) \quad (34)$$

#### 4. The proposed solution

In this paper, a multi-stage algorithm is provided for solving the problem and its flowchart is shown in Fig. 8. The solution is performed in three stages. In the first stage, the TEP problem is performed by using the PSO method. In the second stage, the RPP is performed and in the third stage, the reliability of planning is checked.

##### 4.1. First stage: solving the TEP problem

In this stage, the TEP problem is carried out by using PSO algorithm. Blocks A to H in Fig. 8 show the details of this stage. In block A, all reactive demands are procured locally. In other words, the reactive demands are not considered in this phase of planning. Then, in blocks B to H, the TEP problem is performed by using PSO. In block B, an initial population is generated randomly. In block C, the PDF function of congestion surplus is calculated for each particle in the population. The PDF function is calculated based on the method presented in Fig. 4. It means that, in block C, the flowchart shown in Fig. 4 is performed one time for each particle.

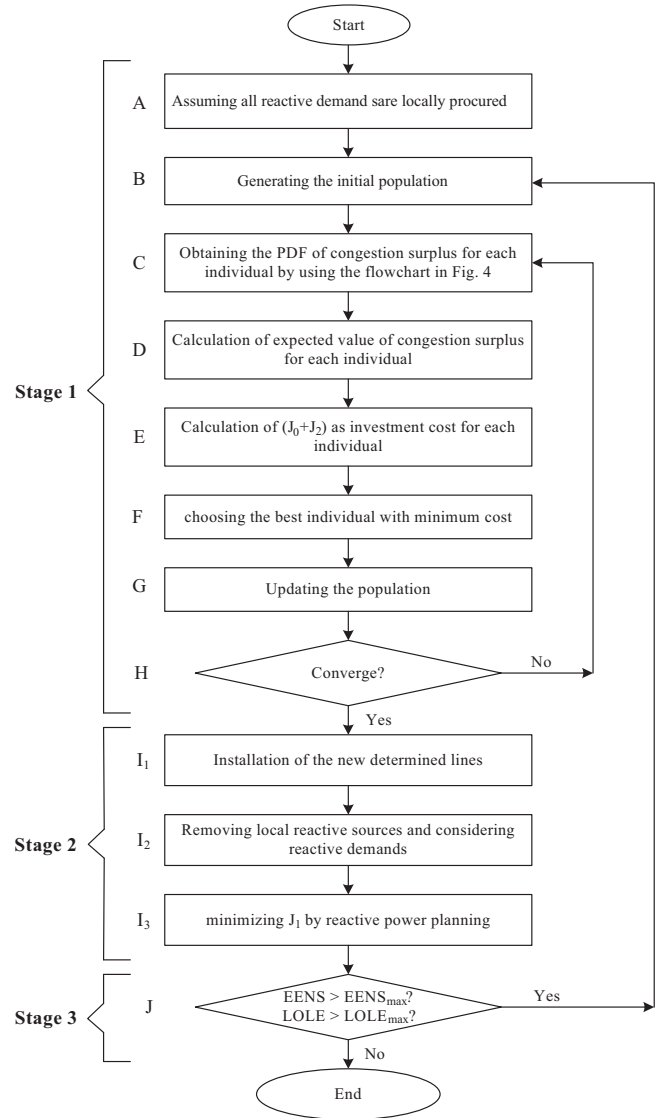


Fig. 8. Flowchart of the proposed planning.

In a population with  $n$  particle, the flowchart in Fig. 4 is performed  $n$  times and  $n$  PDF functions are obtained. Then, in block D the expected value of each PDF function of the congestion surplus is calculated. The investment cost of each particle is computed in block E. Then, in block F, the elite particle with the minimum cost (the sum of investment cost of new lines and the social welfare cost) is chosen. In block G, the PSO population is updated based on the PSO rules and in block H, the stopping criterion is checked which is 15 iterations without changing the best elite. The PSO algorithm is repeated till convergence. After convergence, the new lines are determined. As referred above, at first, all reactive demands are procured locally and hence, the cost of RPP ( $J_1$ ) is not considered in this stage. Therefore, the TEP problem in this stage is carried out with two objective functions. The objectives are the minimization of the investment cost of new lines ( $J_0$ ) and the minimization of the congestion surplus ( $J_2$ ). The sum of these two objectives is considered as the final objective function in this stage. Thus, the proposed TEP problem is formulated as follows:

$$\text{Min } (w_0 J_0 + w_2 J_2) \quad (35)$$

Subject to  
Eqs. (15)–(21) and (23)

The objective function (35) shows a multi-objective optimization problem, where coefficients  $w_0$  and  $w_2$  represent the weighting factors. In this paper,  $J_0$  and  $J_2$  are considered with equal importance and  $w_0 = w_2$ . It should be noted that the weighting factors are chosen based on the importance of parameters and also the system conditions. Since both  $J_0$  and  $J_2$  have same unit “\$/year”, their importance is equal and the same weights are considered for them.

#### 4.2. Second stage: solving the RPP problem

Block I in Fig. 8 represents this stage in details in which RPP is performed by using the PSO method. At first, in block I<sub>1</sub>, the obtained new lines from stage 1 are installed. Then, in block I<sub>2</sub>, the reactive demands are added to the network. Eventually in block I<sub>3</sub>, RPP is performed by using PSO and the feasible locally reactive sources are determined. The RPP is carried out to determine the place of locally reactive sources with the minimum cost. In this paper, RPP is performed to minimize the investment cost of locally reactive sources ( $J_1$ ) and is formulated as the following optimization problem:

$$\text{Min } J_1 \quad (36)$$

Subject to

Eqs. (15)–(22)

At first, in blocks B to H, the TEP problem is solved and then, in block I<sub>3</sub>, the RPP is performed by using PSO.

It should be noted that the output of the wind power is uncertain; therefore the RPP should be incorporated associated with the wind power uncertainties. To deal with this problem and considering the wind power uncertainties in RPP, the presented planning in (36) is performed for the different values of the wind output. The RPP is performed for different values of the wind output which changes from zero to its nominal value. In the proposed method, the output of the wind power changes with 1 MW discrete steps. Eventually, the minimal reactive sources are chosen for installation as locally reactive sources. Fig. 9 shows the RPP output versus the output of the wind power in test case 1. It is seen that the minimum amount of locally reactive sources are obtained, when the output of the wind power is at its maximum rate. Therefore, for any RPP, the output of the wind power is fixed at the maximum power rate and then the RPP is performed.

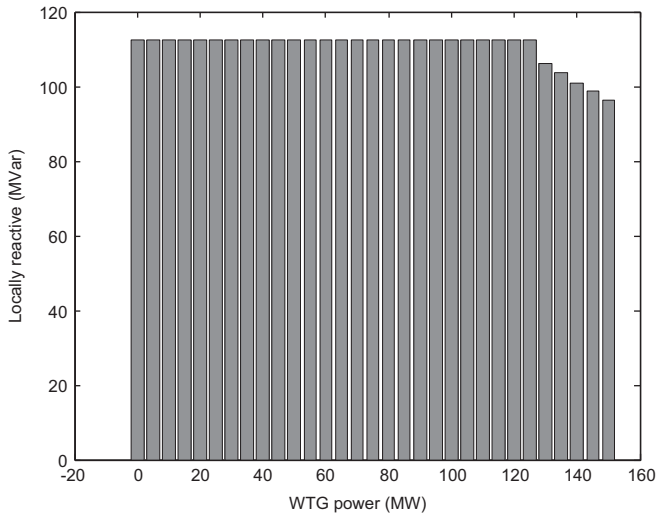


Fig. 9. Locally reactive sources versus WTG output power in test case 1.

#### 4.3. Third stage: reliability assessment

After installation of new lines and new locally reactive sources, the reliability of the system is evaluated in this stage. The resulted planning with EENS or LOLE greater than  $EENS^{max}$  and  $LOLE^{max}$  is eliminated and the algorithm is repeated from the beginning. In this paper, in order to assess the reliability, a constraint is considered for the planning and it is checked in block J. In this block, the reliability of the planning is evaluated by using the reliability indexes EENS and LOLE, which are calculated using (37) and (38).

$$EENS = \frac{\sum_{j=1}^N \text{Total amount of loss of load}}{N_s} \text{ (MWh/year)} \quad (37)$$

$$LOLE = \frac{\sum_{j=1}^N \text{Total hours of loss of load}}{N_s} \text{ (h/Year)} \quad (38)$$

In order to assess the reliability, the combination of OPF and scenario-based MCS is used. In all scenarios of the MCS, the load and WTG output are randomly generated based on their estimated PDF and also a random number is considered for each line outage. For each scenario, the AC-OPF is performed to determine the unsupplied power. The EENS is calculated as the average amount of unsupplied powers for all scenarios.

In this paper, PSO and OPF are used in the planning. Although these two routines are well known, in the next sections, they are briefly explained. The reliability with consideration of DGs is reported in [27].

#### 4.4. Optimal power flow

OPF is a well known tool in power system analysis. It is a nonlinear programming with equality and inequality constraints. It is carried out with different objective functions such as the maximization of the distance to collapse, the maximization of the social welfare etc. Among these objectives, the social welfare maximization is a market oriented objective function. In this paper, a market based OPF is used, which aims at the social welfare maximization (or the congestion surplus minimization). The OPF with the proposed objective function is formulated as follows [28]:

$$\text{Min } J_2 \quad (39)$$

Subject to

Eqs. (15)–(21)

#### 4.5. Updating the population in PSO

As referred before, PSO method is used twice in the proposed planning; at the first stage of the planning, TEP is solved using PSO. In this stage, PSO finds new lines to expansion. In the second stage, RPP is solved using PSO, where the algorithm finds reactive power sources. PSO is a well-known optimization technique which has been widely used to solve different constrained optimization programming. The algorithm has been inspired from social behavior of bird flocking. PSO begins with a random population matrix and it has no evolution operators such as crossover and mutation. The rows in the matrix are called particles. They contain the variable values and are not binary encoded. Each particle moves toward the cost surface with a velocity [29]. In PSO algorithm, each particle is defined as a multi-dimensional particle with two values of  $p_{id}(t)$  and  $v_{id}(t)$ . In each stage of the movement of the swarms, each particle with two best values is updated. Also,  $P_{best}(t)$  and  $g_{best}(t)$  are the local and global best solutions. After finding  $P_{best}(t)$  and  $g_{best}(t)$ , the particles update their velocity and position as

given by (40) and (41);

$$v_{id}(t+1) = w(t)v_{id}(t) + c_1 \text{rand}(p_{best,d}(t) - p_{id}(t)) + c_2 \text{rand}(g_{best,d}(t) - x_{id}(t)) \quad (40)$$

$$p_{id}(t+1) = p_{id}(t) + v_{id}(t+1) \quad (41)$$

where rand is a random value in the range [0,1], the inertia factor  $w$  is used to control the impact of previous velocities on the current velocity during optimization process. The factor  $w$  is linearly decreased from 0.95 to 0.2 [29]. The particles of PSO algorithm usually quickly converge at the initial iterations.

## 5. Illustrative test cases

To show the ability of the proposed methodology, two standard test cases are considered in this paper; the modified Garver 6-bus and the IEEE 24-bus system (RTS). The first test case is a small power system, but the second test case (IEEE 24 bus test system) is a standard and large scale power system provided by IEEE which has been used in many papers as a test system. Generally, the proposed methodology does not have any limitations and can be used for any practical and large scale power systems.

### 5.1. Test case 1

A suitable system for TEP studies is Garver 6-bus system [2]. Since in the system bus 6 is not connected to the rest of the system, it should be connected via new lines. Thus, TEP problem should be incorporated. The basic configuration of this system is depicted in Fig. 10. This system has five buses, six installed lines and 15 candidate lines; its total demands are 760 MW and 152 MVar respectively. In market environment, this system has 10 generating units and five loads [16]. The total generation power at bus 3 is 300 MW, and in this paper, half of this power is considered as WTG with wind speed parameter values:  $V_{ci}=4$ ,  $V_{rate}=10$  and  $V_{co}=22$  m/s. The wind speed has a mean 5.4 m/s and standard deviation 2.7 m/s [21]. It is assumed that the maximum lines between two nodes are five. Also, the fixed and variable costs of reactive sources are  $c_0 = 1000\$$  and  $c_1 = 3 \$/\text{kvar}$ , respectively [3]. The  $EENS^{max}$  is limited by 1%. The project lifetime and the discount rate are considered as 15 years and 10% respectively. Also, the maximum and the minimum voltage levels are 105% and 95%, respectively. Normal distribution with 10% standard deviation is assumed for loads. The FOR and MTTR for all lines are also fixed as 0.03 (fail/year) and 50 (h/year), respectively.

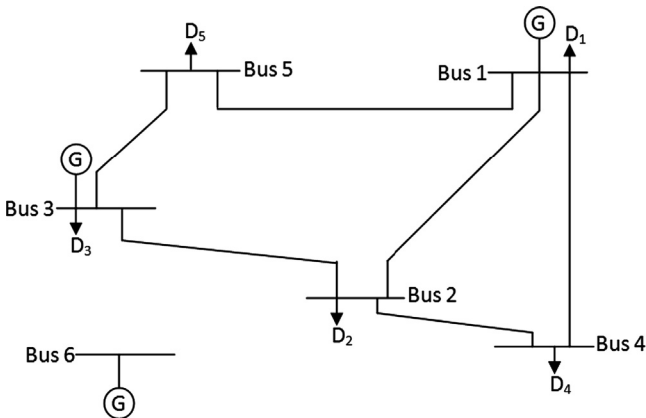


Fig. 10. Modified Graver 6-bus test system.

### 5.2. Test case 2

In order to show the effectiveness of the proposed planning in large scale power systems, the IEEE 24-bus Reliability Test System (RTS) is also used. The system is a well known test system with 24 buses and 41 lines. The system data for power flow and market studies are presented in [4], respectively. Two 180 MW WTG are assumed in buses 1 and 15. For both WTGs, the wind speed parameter values are  $V_{ci}=4$ ,  $V_{rate}=10$  and  $V_{co}=22$  m/s. Also, wind speed has a mean 5.42 m/s and standard deviation 3.06 m/s in bus 1 and the mean value 5.41 m/s and standard deviation 2.7 m/s in bus 15 [21]. The other parameters are considered the same as first test case. It is worth mentioning that all electricity market data have been obtained from the realistic cases of the day-ahead electricity market of mainland Spain based on demand patterns corresponding to year 2004 and all data have been matched with actual values observed in actual markets [4].

## 6. Results and discussions

In this section, the proposed planning is carried out on two test cases. The proposed constrained optimization planning is solved

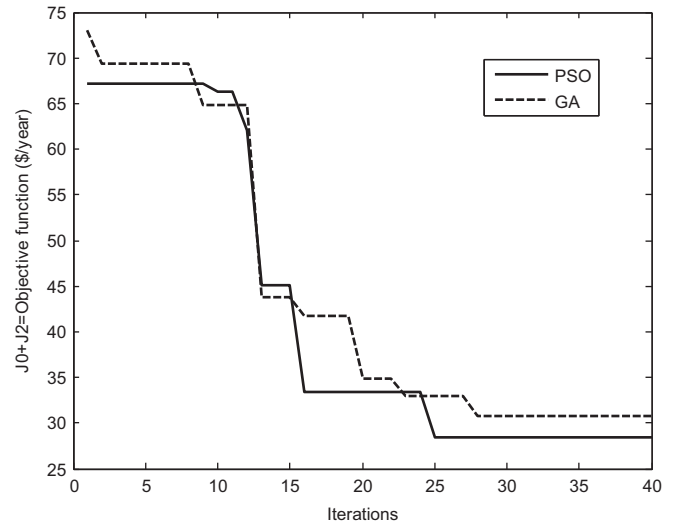


Fig. 11. Convergence of PSO and GA for the first stage of the planning in test case 1.

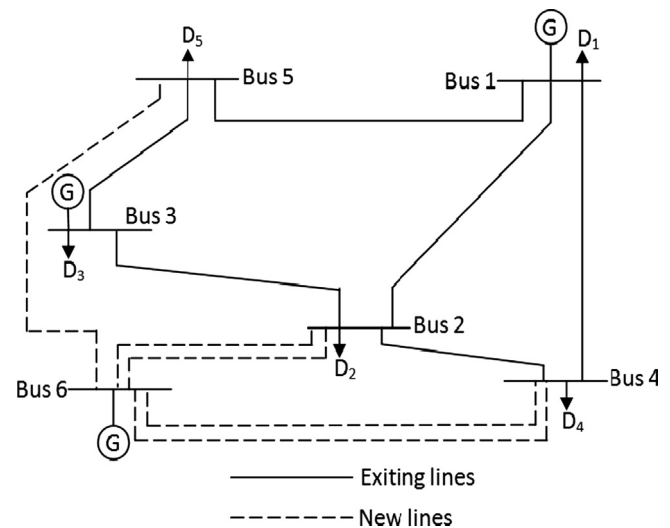


Fig. 12. New determined lines for test case 1.



by using PSO. In order to show the accuracy and the effectiveness of the PSO method, the proposed optimization problem is also solved by GA and results are compared and studied.

### 6.1. Results of test case 1

The proposed planning is carried out on test case 1. The convergence of PSO and GA in solving the first stage of the problem is depicted in Fig. 11. The convergence is obtained after 15 iterations. The system after installation of new lines is depicted in Fig. 12. It can be seen that five new lines are installed. The results of planning are presented in Table 1. The table shows that, PSO-based planning leads to five new lines, while the GA-based planning gives six new lines. In RPP stage, 96.3 MVar is obtained in PSO method, but GA method leads to 85.01 MVar. This means that the GA method leads to less locally

reactive sources than the PSO method. Besides, the congestion surplus in PSO case is 5% greater than GA case. Thus, PSO method leads to fewer new lines, but locally reactive sources and congestion surplus are more than GA method. A complete comparison between GA and PSO is presented in Table 2. This table shows that, the investment cost of new lines in PSO method is  $2.6294 (\$ \times 10^6/\text{year})$  less than GA case. But, the investment cost of locally reactive sources in PSO case is  $0.0044 (\$ \times 10^6/\text{year})$  greater than the GA method. Besides, the congestion surplus of the PSO method is  $0.2268 (\$ \times 10^6/\text{year})$  greater than the GA method. Eventually, the total cost of PSO method is  $2.3983 (\$ \times 10^6/\text{year})$  less than GA method. Regarding all results, it can be concluded that PSO method is a strong optimization technique and can solve the proposed constrained optimization problem better than GA method. PSO method gives a better planning with minimum cost and the obtained plan leads to lower cost while all system constraints

**Table 1**  
Results of planning for test case 1.

	PSO results	GA results
Lines addition in AC-TEP phase	$n_{2-6}=2, n_{4-6}=2, n_{5-6}=1$	$N_{1-5}=1, n_{2-6}=2, n_{4-6}=2, n_{5-6}=1$
VAR source allocation phase	38.36 MVar at bus 4 57.94 MVar at bus 5	41.68 MVar at bus 4 43.33 MVar at bus 5
Expected value of congestion surplus	525.8423 (\$/h) $4.6064 \times 10^6$ (\$/year)	499.9572 (\$/h) $4.3796 \times 10^6$ (\$/year)

**Table 2**  
Comparison of PSO and GA for test case 1.

	Proposed method with PSO	Proposed method with GA
Number of new added lines	5	6
Investment cost of the new lines ( $\$ \times 10^6$ )	181	201
Investment cost of the new lines ( $\$ \times 10^6/\text{year}$ )	23.7968	26.4262
Investment cost of reactive power planning ( $\$ \times 10^6$ )	0.2899	0.2560
Investment cost of reactive power planning ( $\$ \times 10^6/\text{year}$ )	0.0389	0.0345
Congestion surplus (\$/h)	525.8423	499.9572
Congestion surplus ( $\$ \times 10^6/\text{year}$ )	4.6064	4.3796
Total cost of planning ( $\$ \times 10^6/\text{year}$ )	28.4412	30.8395

**Table 3**  
Results of planning for test case 2.

	PSO results	GA results
Lines addition in AC-TEP phase:	$n_{6-10}=2, n_{7-8}=1, n_{11-13}=1, n_{16-17}=1$	$n_{6-10}=2, n_{7-8}=1, n_{11-13}=1, n_{16-17}=1, n_{9-12}=1$
VAR source allocation phase:	206.1 MVar at bus 3 49.41 MVar at bus 4 139.1 MVar at bus 9 573.7 MVar at bus 12 131.2 MVar at bus 24	198.71 MVar at bus 3 29.11 MVar at bus 4 140.2 MVar at bus 9 551.8 MVar at bus 12 128.3 MVar at bus 24
Expected value of congestion surplus	9138 (\$/h) $80.0489 \times 10^6$ (\$/year)	8888 (\$/h) $77.8589 \times 10^6$ (\$/year)

**Table 4**  
Comparison of PSO and GA for test case 2.

	Proposed method with PSO	Proposed method with GA
Number of new added lines	5	6
Investment cost of the new lines ( $\$ \times 10^6$ )	150	200
Investment cost of the new lines ( $\$ \times 10^6/\text{year}$ )	19.7211	26.2948
Investment cost of reactive power planning ( $\$ \times 10^6$ )	3.2991	2.8461
Investment cost of reactive power planning ( $\$ \times 10^6/\text{year}$ )	0.4337	0.3742
Congestion surplus (\$/h)	9138	8888
Congestion surplus ( $\$ \times 10^6/\text{year}$ )	80.0489	77.8589
Total cost of planning ( $\$ \times 10^6/\text{year}$ )	100.2037	104.5278

**Table 5**  
The reliability indexes for both the test cases.

	Test case 1		Test case 2	
	PSO	GA	PSO	GA
EENS (MWh/Year)	136.94	136.04	11158.3	11165.2
LOLE (h/Year)	15.60	19.51	81.18	85.41

are satisfied and also the system performance is guaranteed. Besides, PSO technique gives one line less than the GA method and this result is very favorable for power system operator due to the problems of installing new lines.

### 6.2. Results of test case 2

In this section, the proposed planning is carried out on test case 2. Table 3 shows the planning results. Five new lines are denoted for network expansion in PSO planning, but in GA planning, six new lines are denoted for TEP. In the second stage of planning which is RPP, it is seen that the PSO-TEP denotes 1099.51 MVar locally reactive sources, which is 51 MVar more than the GA method. Besides, the congestion surplus in PSO case is 250 (\$/h) or  $2.19 \times 10^6$  (\$/year) greater than the GA case. Thus, like the first test case, PSO planning gives less new lines and more locally reactive sources and congestion surplus. Therefore, a complete economical comparison of two methods is valuable and presented in Table 4. As seen from this table, the investment cost of new lines in PSO planning is 6.5737 (\$  $\times 10^6$ /year) less than GA planning. But in RPP stage, the investment cost of GA is 0.0595 (\$  $\times 10^6$ /year) less than PSO. Also, the congestion surplus of GA method is 2.19 (\$  $\times 10^6$ /year) less than PSO. Hence, the total cost of planning can be calculated for complete comparison, where, the total cost of the PSO method is 4.3241 (\$  $\times 10^6$ /year) less than the GA method. Considering all results and comparisons, it is very clear to see that PSO technique can be used to solve such constrained optimization programming. PSO method gives an optimal plan with less cost and better performance. It can also be concluded that both PSO and GA methods can solve the problem, but PSO results are more optimal than GA. Eventually with regard to the results, the PSO based plan is chosen as final plan.

### 6.3. Reliability assessment

In long term planning, the reliability assessment is an inevitable parameter. Thus, the reliability of the resulted planning is evaluated by using the most commonly used reliability indexes such as LOLE and EENS. The MCS is chosen for simulation as a suitable tool to deal with uncertainties. Table 5 shows the reliability indexes for both the test cases, respectively. As seen from the table, the reliability of GA-plan and PSO-plan is acceptable and both plans are reliable. However, from the view of the comparison, the PSO technique leads to a plan with less EENS and LOLE, because the obtained plan by PSO comprises less components and its reliability is more.

## 7. Conclusions

In this paper, considering load and wind uncertainties in TEP problem under electricity market was successfully investigated. The presented TEP-RPP problem is a multi-objective optimization problem. In the proposed planning, the social welfare maximization, the investment cost minimization and the reliability improvement were simultaneously obtained. The uncertainties were involved by using MCS and a PDF was obtained for each of them. Also, the reliability assessment was carried out by using MCS. Two optimization methods

were compared and investigated. The simulation results demonstrate that, the PSO method show a significant advantage than the GA method. Also, the proposed method significantly improves the social welfare of the system and resulted in fewer new lines.

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